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Analytic solutions for marginal deformations in open superstring field theory

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Abstract

We extend the calculable analytic approach to marginal deformations recently developed in open bosonic string field theory to open superstring field theory formulated by Berkovits. We construct analytic solutions to all orders in the deformation parameter when operator products made of the marginal operator and the associated superconformal primary field are regular.

1 Introduction

Ever since the analytic solution for tachyon condensation in open bosonic string field theory [1] was constructed by Schnabl [2], new analytic technologies have been developed [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], and analytic solutions for marginal deformations were recently constructed [16, 17].[1] We believe that we are now in a new phase of research on open string field theory.[2]

Extension of these new technologies to closed string field theory, however, does not seem straightforward. The star product [1] used in open string field theory has a simpler description in the conformal field theory (CFT) formulation when we use a coordinate called the sliver frame which was originally introduced in [37]. It has been an important ingredient in recent developments. Closed bosonic string field theory [38, 39, 40, 41, 42, 43] and heterotic string field theory [44, 45], however, use infinitely many non-associative string products, and we have not found any coordinate where simple descriptions of these string products are possible.

On the other hand, extension to open superstring field theory formulated by Berkovits [46] is promising because the string product used in the theory is the same as that in open bosonic string field theory. In this paper we construct analytic solutions for marginal deformations in open superstring field theory.

We first review the solutions for marginal deformations in open bosonic string field theory. The solutions take the form of an expansion in terms of the deformation parameter λ, and analytic expressions to all order in λ have been derived when operator products made of the marginal operator are regular [16, 17]. When the operator product of the marginal operator with itself is singular, solutions were constructed to $O(\lambda^3)$ by regularizing the singularity and by adding counterterms [17].

The goal of this paper is to construct analytic solutions in open superstring field theory when operator products made of the marginal operator and the associated superconformal primary field of dimension 1/2 are regular. It will be a starting point for constructing analytic solutions when these operators have singular operator products. We first simplify the equation of motion for open superstring field theory by field redefinition. We then make an ansatz motivated by the structure of the solutions in the bosonic case and solve the equation of motion analytically. The solutions in the superstring case turn out to be remarkably simple and similar to those in the bosonic case. The final section of the paper is devoted to conclusions and discussion.

We learned that T. Erler independently found analytic solutions for marginal deformations in open superstring field theory [47] prior to our construction.

[1] For earlier study of marginal deformations in string field theory and related work, see [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

[2] See [33, 34, 35, 36] for reviews.

2 Solutions in open bosonic string field theory

In this section, we review the analytic solutions for marginal deformations constructed in [16, 17] for the open bosonic string. The equation of motion for open bosonic string field theory [1] is given by

$$Q_B \Psi + \Psi^2 = 0\,, \tag{2.1}$$

where Ψ is the open string field and Q_B is the BRST operator. All the string products in this paper are defined by the star product [1]. The open bosonic string field Ψ has ghost number 1 and is Grassmann odd. The BRST operator is Grassmann odd and is nilpotent: $Q_B^2 = 0$. It is a derivation with respect to the star product:

$$Q_B\,(\varphi_1\,\varphi_2) = (Q_B \varphi_1)\,\varphi_2 + (-1)^{\varphi_1}\,\varphi_1\,(Q_B\,\varphi_2) \tag{2.2}$$

for any states φ_1 and φ_2, where $(-1)^{\varphi_1} = 1$ when φ_1 is Grassmann even and $(-1)^{\varphi_1} = -1$ when φ_1 is Grassmann odd.

The deformation of the boundary CFT for the open string by a matter primary field V of dimension 1 is marginal to linear order in the deformation parameter. When the deformation is exactly marginal, we expect a solution of the form

$$\Psi_\lambda = \sum_{n=1}^{\infty} \lambda^n\, \Psi^{(n)}\,, \tag{2.3}$$

where λ is the deformation parameter, to the nonlinear equation of motion (2.1). When operator products made of V are regular, analytic expressions of $\Psi^{(n)}$'s were derived in [16, 17], and the BPZ inner product $\langle\, \varphi, \Psi^{(n)}\,\rangle$ for a state φ in the Fock space is given by

$$\begin{aligned}\langle\, \varphi, \Psi^{(n)}\,\rangle = \int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1}\, \langle\, f\circ\varphi(0)\, cV(1)\, \mathcal{B}\, cV(1+t_1)\, \mathcal{B}\, cV(1+t_1+t_2)\, \ldots \\ \times\, \mathcal{B}\, cV(1+t_1+t_2+\ldots+t_{n-1})\,\rangle_{\mathcal{W}_{1+t_1+t_2+\ldots+t_{n-1}}}\,.\end{aligned} \tag{2.4}$$

We follow the notation used in [3, 10, 17]. In particular, see the beginning of section 2 of [3] for the relation to the notation used in [2]. Here and in what follows we use φ to denote a generic state in the Fock space and $\varphi(0)$ to denote its corresponding operator in the state-operator mapping. We use the doubling trick in calculating CFT correlation functions. As in [10], we define the oriented straight lines $V_\alpha^{\pm}$ by

$$\begin{aligned}&V_\alpha^{\pm} = \Big\{\, z \,\Big|\, \mathrm{Re}(z) = \,\pm\,\frac{1}{2}\,(1+\alpha)\,\Big\}\,,\\ &\text{orientation}\;:\; \pm\,\frac{1}{2}\,(1+\alpha) - i\,\infty \to\; \pm\,\frac{1}{2}\,(1+\alpha) + i\,\infty\,,\end{aligned} \tag{2.5}$$

and the surface $\mathcal{W}_\alpha$ can be represented as the region between V_0^- and $V_{2\alpha}^+$, where V_0^- and $V_{2\alpha}^+$ are identified by translation. The function $f(z)$ is

$$f(z) = \frac{2}{\pi} \arctan z \,, \tag{2.6}$$

and $f \circ \varphi(z)$ denotes the conformal transformation of $\varphi(z)$ by the map $f(z)$. The operator $\mathcal{B}$ is defined by

$$\mathcal{B} = \int \frac{dz}{2\pi i} \, b(z) \,, \tag{2.7}$$

and when $\mathcal{B}$ is located between two operators at t_1 and t_2 with $1/2 < t_1 < t_2$, the contour of the integral can be taken to be $-V_\alpha^+$ with $2\,t_1 - 1 < \alpha < 2\,t_2 - 1$. The anticommutation relation of $\mathcal{B}$ and $c(z)$ is

$$\{ \mathcal{B}, c(z) \} = 1 \,, \tag{2.8}$$

and $\mathcal{B}^2 = 0$.

The solution can be written more compactly as

$$\langle\, \varphi, \Psi^{(n)} \,\rangle = \int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1} \Big\langle\, f \circ \varphi(0) \prod_{i=0}^{n-2} \Big[\, cV(1+\ell_i)\, \mathcal{B} \,\Big]\, cV(1+\ell_{n-1}) \,\Big\rangle_{\mathcal{W}_{1+\ell_{n-1}}} \,, \tag{2.9}$$

where

$$\ell_0 = 0 \,, \quad \ell_i \equiv \sum_{k=1}^{i} t_k \quad \text{for} \quad i = 1\,,\,2\,,\,3\,,\,\ldots\,. \tag{2.10}$$

It can be further simplified as

$$\Psi_\lambda = \frac{1}{1 - \lambda\, X_b\, J_b}\, \lambda\, X_b \,, \tag{2.11}$$

where

$$\frac{1}{1 - \lambda\, X_b\, J_b} \equiv 1 + \sum_{n=1}^{\infty} (\, \lambda\, X_b\, J_b \,)^n \,. \tag{2.12}$$

The state X_b is the same as $\Psi^{(1)}$:

$$\langle\, \varphi, X_b \,\rangle = \langle\, f \circ \varphi(0)\, cV(1) \,\rangle_{\mathcal{W}_1} \,. \tag{2.13}$$

It solves the linearized equation of motion: $Q_B X_b = 0$. The definition of J_b is a little involved. It is defined when it appears as $\varphi_1\, J_b\, \varphi_2$ between two states φ_1 and φ_2 in the Fock space. The string product $\varphi_1\, J_b\, \varphi_2$ is given by

$$\langle\, \varphi,\, \varphi_1\, J_b\, \varphi_2 \,\rangle = \int_0^1 dt\, \langle\, f \circ \varphi(0)\, f_1 \circ \varphi_1(0)\, \mathcal{B}\, f_{1+t} \circ \varphi_2(0) \,\rangle_{\mathcal{W}_{1+t}} \,, \tag{2.14}$$

where $\varphi_1(0)$ and $\varphi_2(0)$ are the operators corresponding to the states φ_1 and φ_2, respectively. The map $f_a(z)$ is a combination of $f(z)$ and translation:

$$f_a(z) = \frac{2}{\pi} \arctan z + a \,. \tag{2.15}$$

The string product $\varphi_1 \, J_b \, \varphi_2$ is well defined if $f_1 \circ \varphi_1(0) \, \mathcal{B} \, f_{1+t} \circ \varphi_2(0)$ is regular in the limit $t \to 0$. In the definition of Ψ_λ, J_b always appears between two X_b's. Since $c(1) \, \mathcal{B} \, c(1+t) = c(1)$ in the limit $t \to 0$, the ghost part of $X_b \, J_b \, X_b$ is finite.[3] Therefore, $X_b \, J_b \, X_b$ is well defined if the operator product $V(1) \, V(1+t)$ is regular in the limit $t \to 0$. The ghost part of the state $\Psi^{(n)} = (X_b \, J_b)^{n-1} X_b$ is also finite because $\mathcal{B} \, c(z) \, \mathcal{B} = \mathcal{B}$ and $c(1) \, \mathcal{B} \, c(1 + \ell_{n-1}) = c(1)$ in the limit $\ell_{n-1} \to 0$. Therefore, $\Psi^{(n)}$ is well defined if the operator product in the matter sector

$$\int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1} \prod_{i=0}^{n-1} \Big[\, V(1 + \ell_i) \, \Big] \tag{2.16}$$

is finite. For example, the marginal deformation associated with the rolling tachyon and the deformations in the light-cone directions satisfy the regularity condition [16, 17].

An important property of J_b is

$$\varphi_1 \, (Q_B J_b) \, \varphi_2 = \varphi_1 \, \varphi_2 \tag{2.17}$$

when $f_1 \circ \varphi_1(0) \, f_{1+t} \circ \varphi_2(0)$ vanishes in the limit $t \to 0$. Since the BRST transformation of $b(z)$ is the energy-momentum tensor $T(z)$, the inner product $\langle \, \varphi, \, \varphi_1 \, (Q_B J_b) \, \varphi_2 \, \rangle$ is given by

$$\langle \, \varphi, \, \varphi_1 \, (Q_B J_b) \, \varphi_2 \, \rangle = \int_0^1 dt \, \langle \, f \circ \varphi(0) \, f_1 \circ \varphi_1(0) \, \mathcal{L} \, f_{1+t} \circ \varphi_2(0) \, \rangle_{\mathcal{W}_{1+t}} \,, \tag{2.18}$$

where

$$\mathcal{L} = \int \frac{dz}{2\pi i} \, T(z) \,, \tag{2.19}$$

and the contour of the integral is the same as that of $\mathcal{B}$. As discussed in [3], an insertion of $\mathcal{L}$ is equivalent to taking a derivative with respect to t. It is analogous to the relation $L_0 \, e^{-tL_0} \;=\; - \, \partial_t \, e^{-tL_0}$ in the standard strip coordinates, where L_0 is the zero mode of the energy-momentum tensor. We thus have

$$\begin{aligned} \langle \, \varphi, \, \varphi_1 \, (Q_B J_b) \, \varphi_2 \, \rangle &= \int_0^1 dt \, \partial_t \, \langle \, f \circ \varphi(0) \, f_1 \circ \varphi_1(0) \, f_{1+t} \circ \varphi_2(0) \, \rangle_{\mathcal{W}_{1+t}} \\ &= \langle \, f \circ \varphi(0) \, f_1 \circ \varphi_1(0) \, f_2 \circ \varphi_2(0) \, \rangle_{\mathcal{W}_2} \end{aligned} \tag{2.20}$$

[3] Note that $f_a \circ cV(0) = cV(a)$ because cV is a primary field of dimension 0.

when $f_1 \circ \varphi_1(0)\, f_{1+t} \circ \varphi_2(0)$ vanishes in the limit $t \to 0$. This completes the proof of (2.17). When $\varphi_1 = \varphi_2 = X_b$, the operator product $cV(1)\, cV(1+t)$ vanishes in the limit $t \to 0$ if $V(1)\, V(1+t)$ is regular in the limit $t \to 0$. In the language of [17], $\varphi_1\, J_b\, \varphi_2$ is

$$\varphi_1\, J_b\, \varphi_2 = \int_0^1 dt\, \varphi_1\, e^{-(t-1)L_L^+}\, (-B_L^+)\, \varphi_2\,, \tag{2.21}$$

and the relation (2.17) follows from $\{Q_B,\, B_L^+\} = L_L^+$.

To summarize, when operator products made of V are regular, the solution (2.11) is well defined, and we can safely use the relations

$$Q_B X_b = 0\,, \qquad Q_B J_b = 1 \tag{2.22}$$

for the Grassmann-odd states X_b and J_b when we calculate the BRST transformation of Ψ_λ. It is now straightforward to calculate $Q_B \Psi_\lambda$, and the result is

$$Q_B \Psi_\lambda = -\frac{1}{1-\lambda\, X_b\, J_b}\, \lambda\, X_b\, \frac{1}{1-\lambda\, X_b\, J_b}\, \lambda\, X_b\,. \tag{2.23}$$

We have thus shown that Ψ_λ in (2.11) satisfies the equation of motion (2.1).

3 Equation of motion for open superstring field theory

The equation of motion for open superstring field theory [46] is

$$\eta_0\, (\, e^{-\Phi}\, Q_B\, e^{\Phi}\,) = 0\,, \tag{3.1}$$

where Φ is the open superstring field. It is Grassmann even and has ghost number 0 and picture number 0. The superghost sector is described by η, ξ, and ϕ [48, 49], and the zero modes of η and ξ are included in the Hilbert space. The operator η_0 is the zero mode of η and a derivation with respect to the star product. For any states φ_1 and φ_2, we have

$$\eta_0\, (\varphi_1\, \varphi_2) = (\eta_0\, \varphi_1)\, \varphi_2 + (-1)^{\varphi_1}\, \varphi_1\, (\eta_0\, \varphi_2)\,, \tag{3.2}$$

as in the case of Q_B, where $(-1)^{\varphi_1} = 1$ when φ_1 is Grassmann even and $(-1)^{\varphi_1} = -1$ when φ_1 is Grassmann odd. The Grassmann-odd operator η_0 is nilpotent and anticommutes with Q_B:

$$Q_B^2 = 0\,, \qquad \eta_0^2 = 0\,, \qquad \{Q_B,\, \eta_0\} = 0\,. \tag{3.3}$$

Since $\eta_0\, (\, e^{-\Phi}\, Q_B\, e^{\Phi}\,) = e^{-\Phi}\, [\, Q_B\, (\, e^{\Phi}\, \eta_0\, e^{-\Phi}\,)\,]\, e^{\Phi}$, the equation of motion can also be written as follows:

$$Q_B\, (\, e^{\Phi}\, \eta_0\, e^{-\Phi}\,) = 0\,. \tag{3.4}$$

We further simplify the equation of motion by field redefinition. Since the open superstring field Φ has vanishing ghost and picture numbers, there is a natural class of field redefinitions given by

$$\Phi_{new} = \sum_{n=1}^{\infty} a_n \Phi_{old}^n \,, \tag{3.5}$$

where a_n's are constants. The map from Φ_{old} to Φ_{new} is well defined at least perturbatively. We choose

$$1 - \Phi_{new} = e^{-\Phi_{old}} \,, \tag{3.6}$$

and the equation of motion (3.4) written in terms of Φ_{new} is

$$- Q_B \left(\frac{1}{1-\Phi} \, \eta_0 \, \Phi \right) = - \frac{1}{1-\Phi} \left[\, Q_B \, \eta_0 \, \Phi + (\, Q_B \, \Phi \,) \, \frac{1}{1-\Phi} \, (\, \eta_0 \, \Phi \,) \right] = 0 \,, \tag{3.7}$$

where

$$\frac{1}{1-\Phi} \equiv 1 + \sum_{n=1}^{\infty} \Phi^n \,. \tag{3.8}$$

In the following sections, we solve the equation of motion of the form

$$Q_B \, \eta_0 \, \Phi + (\, Q_B \, \Phi \,) \, \frac{1}{1-\Phi} \, (\, \eta_0 \, \Phi \,) = 0 \,, \tag{3.9}$$

or

$$Q_B \, \eta_0 \, \Phi + (\, Q_B \, \Phi \,) \, (\, \eta_0 \, \Phi \,) + \sum_{n=1}^{\infty} (\, Q_B \, \Phi \,) \, \Phi^n \, (\, \eta_0 \, \Phi \,) = 0 \,. \tag{3.10}$$

4 Solutions to second order

For any marginal deformation of the boundary CFT for the open superstring, there is an associated superconformal primary field $V_{1/2}$ of dimension $1/2$, and the marginal operator V_1 of dimension 1 is the supersymmetry transformation of $V_{1/2}$. For example, $V_{1/2}$ is the fermionic coordinate $\psi^\mu(z)$ when V_1 is the derivative of the bosonic coordinate $i \, \partial X^\mu(z)$ up to a normalization constant. In the RNS formalism, the unintegrated vertex operator in the -1 picture is $ce^{-\phi} V_{1/2}$, and the unintegrated vertex operator in the 0 picture is cV_1. In open superstring field theory [46], the solution to the linearized equation of motion $Q_B \, \eta_0 \, \Phi^{(1)} = 0$ associated with the marginal deformation is given by $\Phi^{(1)} = X$, where X is the state corresponding to the operator $\mathcal{V}(0) = c \, \xi e^{-\phi} V_{1/2}(0)$:

$$\langle \, \varphi, X \, \rangle = \langle \, f \circ \varphi(0) \, \mathcal{V}(1) \, \rangle_{\mathcal{W}_1} = \langle \, f \circ \varphi(0) \, c \, \xi e^{-\phi} V_{1/2}(1) \, \rangle_{\mathcal{W}_1} \,. \tag{4.1}$$

See [28] for some explicit calculations in open superstring field theory when $V_{1/2}(z) = \psi^\mu(z)$.

When the deformation is exactly marginal, we expect a solution of the form

$$\Phi_\lambda = \sum_{n=1}^{\infty} \lambda^n \, \Phi^{(n)} \,, \tag{4.2}$$

where λ is the deformation parameter, to the nonlinear equation of motion (3.9). The equation for $\Phi^{(2)}$ is

$$Q_B \, \eta_0 \, \Phi^{(2)} = \, - \, (Q_B \, \Phi^{(1)}) \, (\eta_0 \, \Phi^{(1)}) = \, - \, (Q_B X) \, (\eta_0 X) \, . \tag{4.3}$$

The right-hand side is annihilated by Q_B and by η_0 because $Q_B \eta_0 X = 0$. In order to solve the equation for $\Phi^{(2)}$, we introduce a state J by replacing $b(z)$ in J_b for the bosonic case with $\xi b(z)$. Since

$$\eta_0 \cdot \xi b(z) \equiv \oint \frac{dw}{2\pi i} \, \eta(w) \, \xi b(z) = b(z) \tag{4.4}$$

and the BRST transformation of $b(z)$ gives the energy-momentum tensor, we expect that $\xi b(z)$ in the superstring case plays a similar role of $b(z)$ in the bosonic case. In fact, the zero mode of $\xi b(z)$ divided by L_0 was used in the calculation of on-shell four-point amplitudes in [50]. We again define J when it appears as $\varphi_1 \, J \, \varphi_2$ between two states φ_1 and φ_2 in the Fock space. The string product $\varphi_1 \, J \, \varphi_2$ is given by

$$\langle \, \varphi, \, \varphi_1 \, J \, \varphi_2 \, \rangle = \int_0^1 dt \, \langle \, f \circ \varphi(0) \, f_1 \circ \varphi_1(0) \, \mathcal{J} \, f_{1+t} \circ \varphi_2(0) \, \rangle_{\mathcal{W}_{1+t}} \,, \tag{4.5}$$

where $\varphi_1(0)$ and $\varphi_2(0)$ are the operators corresponding to the states φ_1 and φ_2, respectively. The operator $\mathcal{J}$ is defined by

$$\mathcal{J} = \int \frac{dz}{2\pi i} \, \xi b(z) \,, \tag{4.6}$$

and when $\mathcal{J}$ is located between two operators at t_1 and t_2 with $1/2 < t_1 < t_2$, the contour of the integral can be taken to be $-V_\alpha^+$ with $2\,t_1 - 1 < \alpha < 2\,t_2 - 1$. As in the case of J_b, the string product $\varphi_1 \, J \, \varphi_2$ is well defined if $f_1 \circ \varphi_1(0) \, \mathcal{J} \, f_{1+t} \circ \varphi_2(0)$ is regular in the limit $t \to 0$. We also have an important relation

$$\varphi_1 \, (\, Q_B \, \eta_0 \, J \,) \, \varphi_2 = \varphi_1 \, \varphi_2 \tag{4.7}$$

if $f_1 \circ \varphi_1(0) \, f_{1+t} \circ \varphi_2(0)$ vanishes in the limit $t \to 0$. The proof of this relation follows from that of (2.17) after we use (4.4) in calculating $\eta_0 J$. We will discuss these regularity conditions later and proceed for the moment assuming they are satisfied. Namely, we assume that states involving J are well defined and that we can use the relations

$$Q_B \eta_0 X = 0 \,, \qquad Q_B \eta_0 J = 1 \tag{4.8}$$

for the Grassmann-even states X and J.

Motivated by the structure of the solutions in the bosonic case, we look for a solution which consists of $X\,J\,X$, Q_B, and η_0 to the equation (4.3) for $\Phi^{(2)}$. There are nine possible states:

$$\begin{array}{lll} (Q_B\eta_0 X)\,J\,X = 0\,, & (Q_B X)\,(\eta_0 J)\,X\,, & (Q_B X)\,J\,(\eta_0 X)\,, \\ (\eta_0 X)\,(Q_B J)\,X\,, & X\,(Q_B\eta_0 J)\,X = X^2\,, & X\,(Q_B J)\,(\eta_0 X)\,, \\ (\eta_0 X)\,J\,(Q_B X)\,, & X\,(\eta_0 J)\,(Q_B X)\,, & X\,J\,(Q_B\eta_0 X) = 0\,. \end{array} \tag{4.9}$$

Two of them vanish and one of them reduces to X^2. We then calculate the action of $Q_B\eta_0$ on the nonvanishing states:

$$\begin{aligned} Q_B\eta_0\,[\,(Q_B X)\,(\eta_0 J)\,X\,] &= -\,(Q_B X)\,(\eta_0 X)\,, \\ Q_B\eta_0\,[\,(Q_B X)\,J\,(\eta_0 X)\,] &= (Q_B X)\,(\eta_0 X)\,, \\ Q_B\eta_0\,[\,(\eta_0 X)\,(Q_B J)\,X\,] &= -\,(\eta_0 X)\,(Q_B X)\,, \\ Q_B\eta_0\,[\,X\,(Q_B\eta_0 J)\,X\,] &= -\,(\eta_0 X)\,(Q_B X) + (Q_B X)\,(\eta_0 X)\,, \\ Q_B\eta_0\,[\,X\,(Q_B J)\,(\eta_0 X)\,] &= -\,(Q_B X)\,(\eta_0 X)\,, \\ Q_B\eta_0\,[\,(\eta_0 X)\,J\,(Q_B X)\,] &= (\eta_0 X)\,(Q_B X)\,, \\ Q_B\eta_0\,[\,X\,(\eta_0 J)\,(Q_B X)\,] &= -\,(\eta_0 X)\,(Q_B X)\,. \end{aligned} \tag{4.10}$$

We thus find that $(Q_B X)\,(\eta_0 J)\,X$, $-\,(Q_B X)\,J\,(\eta_0 X)$, and $X\,(Q_B J)\,(\eta_0 X)$ solve the equation (4.3) for $\Phi^{(2)}$. We can also take an appropriate linear combination of the seven states, and different solutions should be related by gauge transformations. We choose

$$\Phi^{(2)} = (Q_B X)\,(\eta_0 J)\,X \tag{4.11}$$

and consider its extension to $\Phi^{(n)}$ in the next section.

5 Solutions in open superstring field theory

Remarkably, a simple extension of $\Phi^{(2)}$ in (4.11) solves the equation of motion (3.9) to all orders in λ. A solution is given by

$$\begin{aligned} \Phi^{(3)} &= (Q_B X)\,(\eta_0 J)\,(Q_B X)\,(\eta_0 J)\,X\,, \\ \Phi^{(4)} &= (Q_B X)\,(\eta_0 J)\,(Q_B X)\,(\eta_0 J)\,(Q_B X)\,(\eta_0 J)\,X\,, \\ &\;\;\vdots \\ \Phi^{(n)} &= [\,(Q_B X)\,(\eta_0 J)\,]^{n-1}\,X\,, \end{aligned} \tag{5.1}$$

or

$$\Phi_\lambda = \frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,X\,, \tag{5.2}$$

where

$$\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)} \equiv 1+\sum_{n=1}^{\infty}\,[\,\lambda\,(Q_B X)\,(\eta_0 J)\,]^n\,. \tag{5.3}$$

Let us now show that Φ_λ given by (5.2) satisfies the equation of motion (3.9). Since $Q_B X$ and $\eta_0 J$ are annihilated by η_0, the state $\eta_0\,\Phi_\lambda$ is given by

$$\eta_0\,\Phi_\lambda = \frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,(\eta_0 X)\,. \tag{5.4}$$

For the calculation of $Q_B\,\Phi_\lambda$, we use $Q_B\,[\,(Q_B X)\,(\eta_0 J)\,] = \,-\,Q_B X$ to find

$$Q_B\,\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)} = \,-\,\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,(Q_B X)\,\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,. \tag{5.5}$$

The state $Q_B\,\Phi_\lambda$ is given by

$$\begin{aligned} Q_B\,\Phi_\lambda = &-\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,(Q_B X)\,\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,X \\ &+\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,(Q_B X) \\ =&\,\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,(Q_B X)\,\Big[\,1-\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,X\,\Big]\,. \end{aligned} \tag{5.6}$$

Note that

$$(\,Q_B\,\Phi_\lambda\,)\,\frac{1}{1-\Phi_\lambda} = \frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,(Q_B X)\,. \tag{5.7}$$

Finally, $Q_B\,\eta_0\Phi_\lambda$ is given by

$$Q_B\,\eta_0\,\Phi_\lambda = \,-\,\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,(Q_B X)\,\frac{1}{1-\lambda\,(Q_B X)\,(\eta_0 J)}\,\lambda\,(\eta_0 X)\,. \tag{5.8}$$

We have thus shown that Φ_λ given by (5.2) satisfies the equation of motion (3.9).

An explicit expression of $\Phi^{(n)}$ in the CFT formulation is given by

$$\langle\,\varphi, \Phi^{(n)}\,\rangle = \int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1}\,\Big\langle\,f\circ\varphi(0)\,\prod_{i=0}^{n-2}\Big[\,Q_B\cdot\mathcal{V}(1+\ell_i)\,\mathcal{B}\,\Big]\,\mathcal{V}(1+\ell_{n-1})\,\Big\rangle_{\mathcal{W}_{1+\ell_{n-1}}}\,, \tag{5.9}$$

where the BRST transformation of $\mathcal{V}$ is

$$Q_B\cdot\mathcal{V}(z) = cV_1(z)+\eta e^{\phi}V_{1/2}(z)\,. \tag{5.10}$$

Note that $\mathcal{J}$ in J has been replaced by $\mathcal{B}$ in $\eta_0 J$ because of (4.4). The term $\eta e^{\phi}V_{1/2}(1+\ell_i)$ in $Q_B\cdot\mathcal{V}(1+\ell_i)$ does not contribute when $i = 1,\,2,\,\ldots\,, n-2$ because $\mathcal{B}^2 = 0$. By repeatedly

using $\mathcal{B}\, c(z)\, \mathcal{B} = \mathcal{B}$, we find

$$\begin{aligned} \langle\, \varphi, \Phi^{(n)}\, \rangle = & \int d^{n-1}t \Big\langle\, f\circ\varphi(0)\ cV_1(1)\, \mathcal{B} \prod_{i=1}^{n-2} \Big[\, V_1(1+\ell_i)\, \Big]\, c\,\xi e^{-\phi} V_{1/2}(1+\ell_{n-1})\, \Big\rangle_{\mathcal{W}_{1+\ell_{n-1}}} \\ & + \int d^{n-1}t \Big\langle\, f\circ\varphi(0)\ \eta e^{\phi} V_{1/2}(1)\, \mathcal{B} \prod_{i=1}^{n-2} \Big[\, V_1(1+\ell_i)\, \Big]\, c\,\xi e^{-\phi} V_{1/2}(1+\ell_{n-1})\, \Big\rangle_{\mathcal{W}_{1+\ell_{n-1}}}, \end{aligned} \tag{5.11}$$

where we have defined

$$\int d^{n-1}t \equiv \int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1}\,. \tag{5.12}$$

We can also construct a different solution if we choose $\Phi^{(2)}$ to be $X\,(Q_B J)\,(\eta_0 X)$. It is easy to show that $\overline{\Phi}_\lambda$ given by

$$\overline{\Phi}_\lambda = \lambda\, X\, \frac{1}{1-\lambda\,(Q_B J)\,(\eta_0 X)} \tag{5.13}$$

satisfies the equation of motion (3.9). It is also straightforward to construct analytic solutions based on star-algebra projectors other than the sliver state using the method in [10].

6 Regularity conditions

In the proof that the solution (5.2) satisfies the equation of motion (3.9), we used the following relations:

$$\begin{aligned} (Q_B X)\,(Q_B \eta_0 J)\, X &= (Q_B X)\, X\,, \\ (Q_B X)\,(Q_B \eta_0 J)\,(Q_B X) &= (Q_B X)\,(Q_B X)\,, \\ (Q_B X)\,(Q_B \eta_0 J)\,(\eta_0 X) &= (Q_B X)\,(\eta_0 X)\,. \end{aligned} \tag{6.1}$$

Let us study the conditions for these relations to hold. Since

$$\begin{aligned} \eta_0 \cdot \mathcal{V}(z) &= \eta_0 \cdot [\, c\xi e^{-\phi} V_{1/2}(z)\,] = -ce^{-\phi} V_{1/2}(z)\,, \\ Q_B \cdot \mathcal{V}(z) &= Q_B \cdot [\, c\xi e^{-\phi} V_{1/2}(z)\,] = cV_1(z) + \eta e^{\phi} V_{1/2}(z)\,, \end{aligned} \tag{6.2}$$

and $\mathcal{V}$, $Q_B \cdot \mathcal{V}$, and $\eta_0 \cdot \mathcal{V}$ are all primary fields of dimension 0, the condition for (4.7) gives

$$\begin{aligned} \lim_{w\to z}\, [\, cV_1(z) + \eta e^{\phi} V_{1/2}(z)\,]\, c\xi e^{-\phi} V_{1/2}(w) &= 0\,, \\ \lim_{w\to z}\, [\, cV_1(z) + \eta e^{\phi} V_{1/2}(z)\,]\,[\, cV_1(w) + \eta e^{\phi} V_{1/2}(w)\,] &= 0\,, \\ \lim_{w\to z}\, [\, cV_1(z) + \eta e^{\phi} V_{1/2}(z)\,]\, ce^{-\phi} V_{1/2}(w) &= 0\,. \end{aligned} \tag{6.3}$$

These are satisfied if the operator products $V_1(z)\, V_{1/2}(w)$ and $V_1(z)\, V_1(w)$ are regular in the limit $w \to z$, and $V_{1/2}(z)\, V_{1/2}(w)$ vanishes in the limit $w \to z$. The vertex operator $V_{1/2}(z)$ is

Grassmann odd so that the last condition is satisfied if the operator product $V_{1/2}(z)\,V_{1/2}(w)$ is not singular. To summarize, the equation of motion is satisfied if the operator products $V_1(z)\,V_{1/2}(w)$, $V_1(z)\,V_1(w)$, and $V_{1/2}(z)\,V_{1/2}(w)$ are regular in the limit $w \to z$.

Let us next consider if the solution itself is finite and if any intermediate steps in the proof are well defined. The expressions can be divergent when two or more operators collide, but if the states

$$[\,(Q_B X)\,(\eta_0 J)\,]^{n-1}\,X\,, \qquad [\,(Q_B X)\,(\eta_0 J)\,]^{n-1}\,(Q_B X)\,, \qquad [\,(Q_B X)\,(\eta_0 J)\,]^{n-1}\,(\eta_0 X) \tag{6.4}$$

for any positive integer n are finite, the solution and any intermediate steps in the proof are well defined. An explicit expression of $\Phi^{(n)} = [\,(Q_B X)\,(\eta_0 J)\,]^{n-1}\,X$ has been presented in (5.11). Expressions of $[\,(Q_B X)\,(\eta_0 J)\,]^{n-1}\,(Q_B X)$ and $[\,(Q_B X)\,(\eta_0 J)\,]^{n-1}\,(\eta_0 X)$ can be obtained from (5.11) by replacing $c\,\xi e^{-\phi} V_{1/2}(1+\ell_{n-1})$ with $cV_1(1+\ell_{n-1}) + \eta e^{\phi} V_{1/2}(1+\ell_{n-1})$ and with $-ce^{-\phi} V_{1/2}(1+\ell_{n-1})$, respectively. The bc ghost sector is finite because $c(z)\,\mathcal{B}\,c(w)$ is finite in the limit $w \to z$. The superghost sector is also finite because $\eta e^{\phi}(1)\,\xi e^{-\phi}(1+\ell_{n-1})$ and $\eta e^{\phi}(1)\,\eta e^{\phi}(1+\ell_{n-1})$ are finite in the limit $\ell_{n-1} \to 0$. Therefore, all the expressions are well defined if the contributions from the matter sector listed below are finite:

$$\begin{aligned}
&\int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1}\; \prod_{i=0}^{n-1} \Big[\, V_1(1+\ell_i) \,\Big]\,, \\
&\int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1}\; V_{1/2}(1)\, \prod_{i=1}^{n-1} \Big[\, V_1(1+\ell_i) \,\Big]\,, \\
&\int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1}\; \prod_{i=0}^{n-2} \Big[\, V_1(1+\ell_i) \,\Big]\, V_{1/2}(1+\ell_{n-1})\,, \\
&\int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1}\; V_{1/2}(1)\, \prod_{i=1}^{n-2} \Big[\, V_1(1+\ell_i) \,\Big]\, V_{1/2}(1+\ell_{n-1})\,,
\end{aligned} \tag{6.5}$$

where ℓ_i was defined in (2.10). To summarize, if operator products of an arbitrary number of V_1's and at most two $V_{1/2}$'s are regular, the solution (5.2) is well defined and satisfies the equation of motion (3.9).

7 Conclusions and discussion

We have constructed analytic solutions for marginal deformations in open superstring field theory when operator products made of V_1's and $V_{1/2}$'s are regular. Our solutions are very simple and remarkably similar to the solutions in the bosonic case [16, 17]. We expect that there will be further progress of analytic methods in open superstring field theory.

It would be interesting to study the rolling tachyon in open superstring field theory, and we expect that marginal deformations associated with the rolling tachyon solutions satisfy the regularity conditions discussed in the preceding section. However, deformations we are interested in typically have singular operator products of the marginal operator. In the bosonic case, solutions to third order in λ have been constructed when the operator product of the marginal operator is singular [17]. We hope that a procedure similar to the one developed in the bosonic case will work in the superstring case, and it is important to carry out the program to all orders in the deformation parameter.

Our choice of $\Phi^{(2)}$ in (4.11) was based on a technical reason, and it is not clear if this gauge choice is physically suitable. In particular, the solution Φ_λ in (5.2) does not satisfy the reality condition on the string field. However, it is difficult for us to imagine that there are two inequivalent solutions generated by a single marginal operator which coincide to linear order in λ, and we expect that our solution is related to a real one by a gauge transformation. In fact, we can explicitly confirm this at $O(\lambda^2)$. In order to see this, it is useful to write the solution in the original definition of the string field by inverting the field redefinition (3.6):

$$\Phi_{old} = -\ln(1 - \Phi_{new}) = \sum_{n=1}^{\infty} \frac{1}{n} \Phi_{new}^n . \tag{7.1}$$

We expand Φ_{old} in powers of λ as

$$\Phi_{old} = \sum_{n=1}^{\infty} \lambda^n \Phi_{old}^{(n)} , \tag{7.2}$$

and then $\Phi_{old}^{(2)}$ is given by

$$\Phi_{old}^{(2)} = \Phi_{new}^{(2)} + \frac{1}{2} (\Phi_{new}^{(1)})^2 = (Q_B X)(\eta_0 J) X + \frac{1}{2} X^2 . \tag{7.3}$$

The string field $\Phi_{old}^{(2)}$ does not satisfy the reality condition.[4] However, there is another solution which satisfies the reality condition given by

$$\frac{1}{2} [(Q_B X)(\eta_0 J) X + X (\eta_0 J)(Q_B X)] , \tag{7.4}$$

and the difference between (7.3) and (7.4) is

$$(Q_B X)(\eta_0 J) X + \frac{1}{2} X^2 - \frac{1}{2} [(Q_B X)(\eta_0 J) X + X (\eta_0 J)(Q_B X)] = \frac{1}{2} Q_B [X (\eta_0 J) X] \tag{7.5}$$

[4] A string field within our ansatz satisfies the reality condition when it is odd under the conjugation given by replacing $X \to -X$ and by reversing the order of string products. Signs from anticommuting Grassmann-odd string fields have to be taken care of in reversing the order of string products.

and can be eliminated by a gauge transformation. The open superstring field theory formulated by Berkovits can also be used to describe the $N = 2$ string by replacing Q_B and η_0 with the generators in the $N = 2$ string [46], but the reality condition on the string field for the $N = 2$ string does not seem to be satisfied for Φ_λ in (5.2) either.[5] The conjugation in [46] seems to map Φ_λ in (5.2) to $\overline{\Phi}_\lambda$ in (5.13). We again expect that our solution is related to a solution satisfying the reality condition by a gauge transformation. For example, $-(Q_B X)\, J\, (\eta_0 X)$, which is another solution to the equation for $\Phi^{(2)}$, seems to satisfy the reality condition, and the difference between $-(Q_B X)\, J\, (\eta_0 X)$ and $\Phi^{(2)}$ in (4.11) is $\eta_0\, [\, (Q_B X)\, J\, X\,]$ and can be eliminated by a gauge transformation generated by η_0. We have also found that $(Q_B X)\, (Q_B J)\, X\, (\eta_0 J)\, (\eta_0 X)$, which seems to satisfy the reality condition, solves the equation for $\Phi^{(3)}$ when $\Phi^{(2)}$ is $-(Q_B X)\, J\, (\eta_0 X)$, but we have not been able to extend the solution to all orders in λ. We think that there is a good chance that solutions satisfying the reality condition for the ordinary superstring or for the $N = 2$ string can be found within our ansatz, and it would be desirable to have their explicit expressions. On the other hand, we believe that the solution in (5.2) has an advantage because the actions of Q_B and η_0 on (5.2) are very simple.

It has been expected that the moduli space of D-branes are reproduced by the moduli space of solutions to open string field theory, and we think that our approach provides a concrete setup to address this question. We have seen a one-to-one correspondence between the condition for exact marginality in boundary CFT [51] and the absence of obstruction in solving the equation of motion for string field theory at $O(\lambda^2)$ in the bosonic case [17]. It would be important to study the correspondence at higher orders and in the superstring case, and a better understanding of the correspondence might help us complete the program of constructing solutions when the operator product of the marginal operator is singular. We hope that further developments in this subject will shed light on more conceptual issus in string theory such as background independence or the question why the condition that the β function vanishes in the world-sheet theory gives the equation of motion in the spacetime theory.

Note added

After the first version of this paper was submitted to arXiv, we found analytic solutions satisfying the reality condition [52]. We also learned that T. Erler independently constructed analytic solutions satisfying the reality condition, which were presented in the second version of [47].

[5] Our understanding is that the conjugation in [46] is given by replacing $X \to X$, $J \to -J$, $Q_B \to \eta_0$, and $\eta_0 \to Q_B$ and by reversing the order of string products, and the string field should be even under the conjugation. Again signs from anticommuting Grassmann-odd string fields have to be taken care of in reversing the order of string products. The string field Φ_{new} in (3.6) is real when Φ_{old} is real with respect to this reality condition, while this is not the case for the reality condition for the ordinary superstring discussed earlier.

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